#### CS 589 Fall 2020

# Probability ranking principle

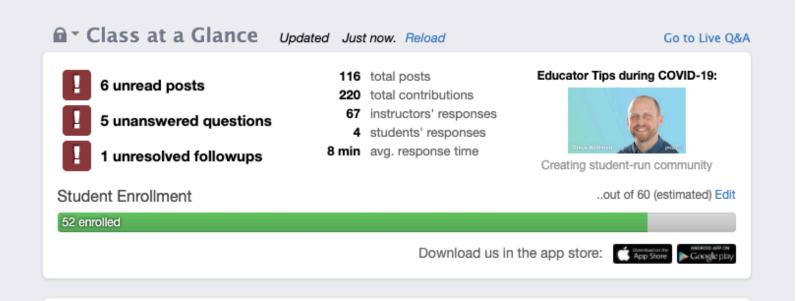
# **Probabilistic retrieval models**

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#### Piazza

- Only 2 students had enrolled in Piazza
- Therefore, I have to state our requirement again
  - Outside of OH, if you choose to use email, your question will be answered significantly slower, email: 2 days



# **Recap of last lecture**

- The boolean retrieval system
- Vector-space model
  - TF: representing documents/queries with a term-document matrix
- Rescaling methods:
  - IDF: penalizing words which appears everywhere
  - Term frequency rescaling (logarithmic, max normalization)
  - Pivoted length normalization

### **Question from last lecture**

 Between the two term-frequency rescaling methods, which one works better? Max normalization or logarithmic?

Max TF $tf(w,d) = \alpha + (1-\alpha) \frac{count(w,d)}{max_v count(v,d)}$ normalization

- Max TF is unstable:
  - max TF in a document vary with change of stop words set
  - When max TF in document d is an outlier, the normalization is incomparable with other documents
  - Does not work well with documents with different TF distribution

### **Today's lecture**

- Basic statistics knowledge
  - Random variables, Bayes rules, maximum likelihood estimation
- Probabilistic ranking principle
- Probability retrieval models
  - Robertson & Spark Jones model (RSJ model)
  - BM25 model
  - Language model based retrieval model

# **Quiz from last lecture**

- Suppose we have one query and two documents:
  - q = "covid 19"
  - doc1 = "covid patient"
  - doc2 = "19 99 car wash"
  - doc3 = "19 street covid testing facility is reopened next week"
- What are the rankings of score(q, doc) using VS model (w/o IDF)?
  - A. doc1 > doc2 > doc3
  - B. doc1 = doc3 > doc2
  - C. doc1 > doc3 > doc2
  - D. doc3 > doc1 > doc2

#### Answer

• Recall the VS model:

$$score(q, d) = \frac{q \cdot d}{\|q\| \cdot \|d\|}$$

- q = "covid 19"
- doc1 = "covid patient"
- doc2 = "19 99 car wash"
- doc3 = "19 street covid testing facility is reopen next week"
- score(q, doc1) = 1/sqrt(2)/sqrt(2) = 0.4999, score(q, doc2) = 1/sqrt(2)/sqrt(4) = 0.3535, score(q, doc3) = 2/sqrt(2)/sqrt(9) = 0.4714
- Therefore the answer is C: doc1 > doc3 > doc2

# **Random variables**

• Random variables



 sequence = 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0
 Observation

  $p(up) = \alpha, p(down) = 1 - \alpha$   $\alpha : parameter$ 

$$p(sequence) = \alpha \times (1 - \alpha) \cdots \times (1 - \alpha) \times (1 - \alpha)$$
$$= \alpha^{\#up} \times (1 - \alpha)^{\#down}$$

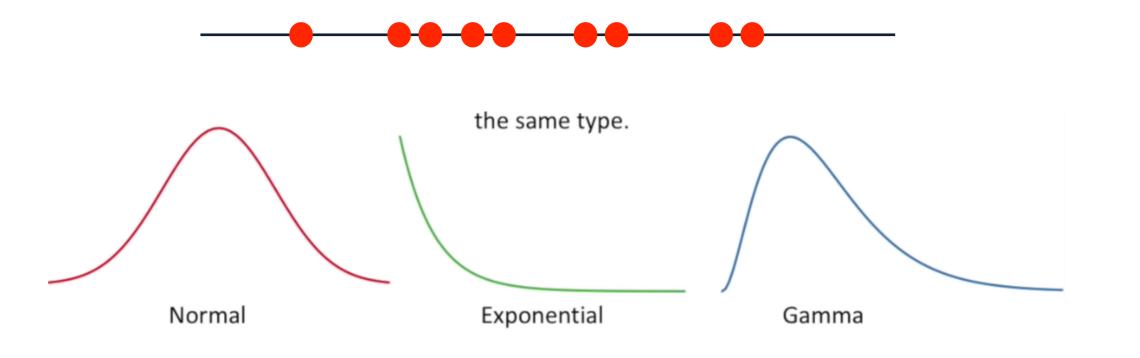
$$\Rightarrow \alpha = \frac{\#up}{\#up + \#down}$$

**Bernoulli distribution** 

#### **Maximum likelihood estimation**

### **Maximum likelihood estimation**

- Fitting a distribution model to the data
  - Assumes mouse weights follow an underlying distribution



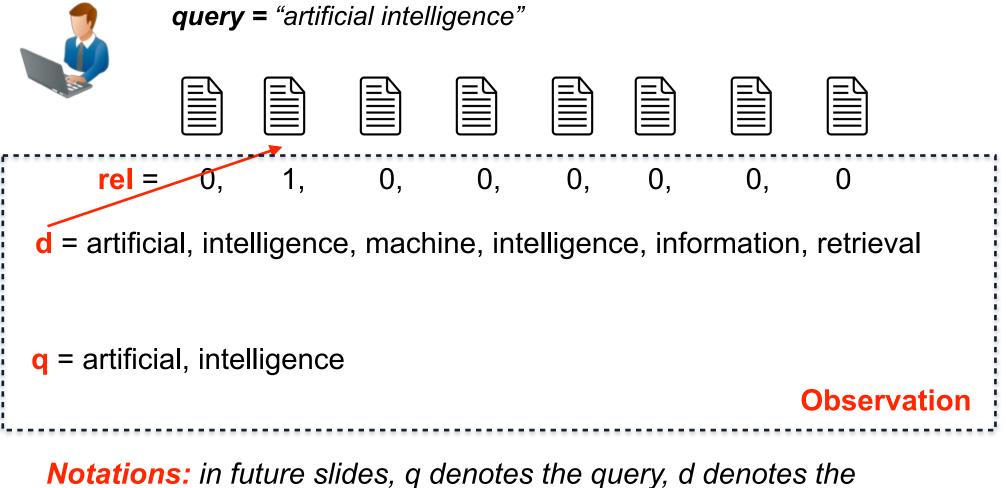
# Maximum likelihood estimation

- Fitting a distribution to the data
  - Distributions of mouse weights



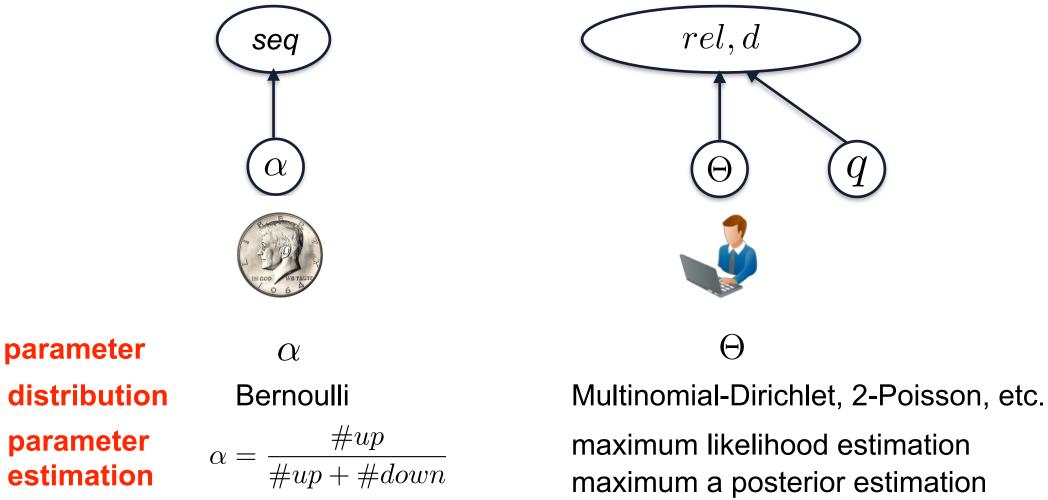
- Applications
  - Making estimations for probabilities for future events to happen
  - For example, predicting the probability for a document to be relevant to a query, and rank all documents by their estimated relevance score

### **Random variables in information retrieval**



document, rel denotes the relevance judgment

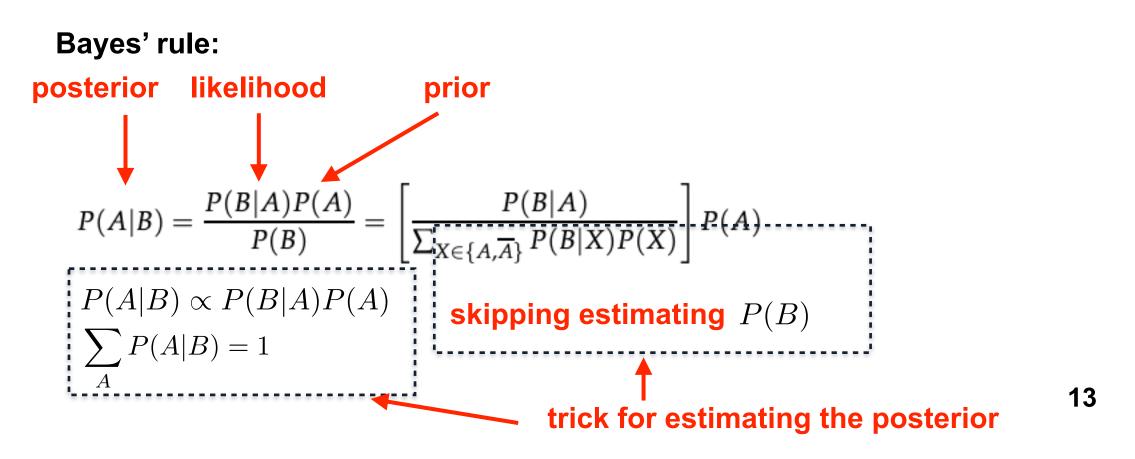
# Probabilistic graphical model (underlying distribution)



#### **Bayes' rules**

#### Chain rule: joint distribution

 $P(A,B) = P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$ 



### **Probability ranking principle**

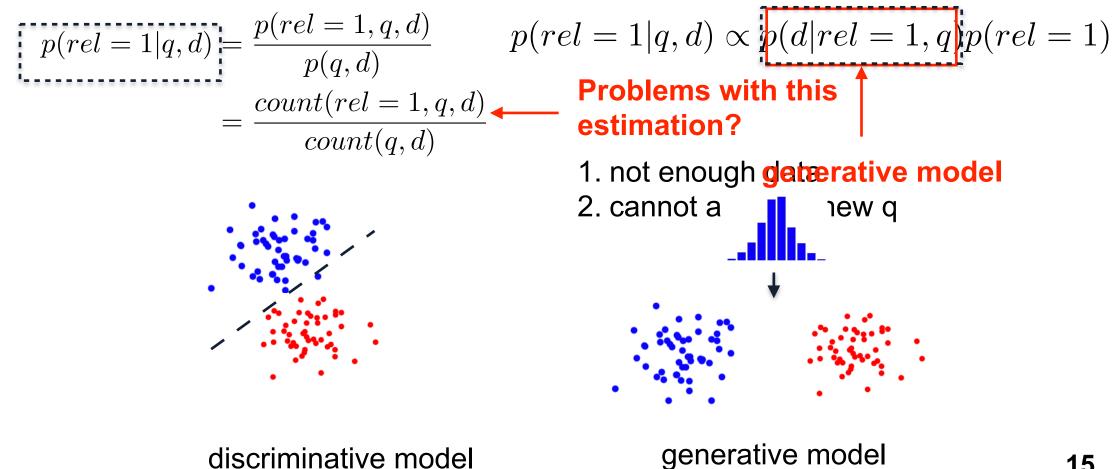
 Assume documents are labelled by 0/1 labels (i.e., the relevance judgement is either 0 or 1), given query q, documents should be ranked on their probabilities of relevance (van Rijsbergen 1979):

**PRP:** rank documents by p(rel = 1|q, d)

• **Theorem**. The PRP is optimal, in the sense that it minimizes the expected loss (Ripley 1996)

**Notations:** in future slides, q denotes the query, d denotes the document, rel denotes the relevance judgment

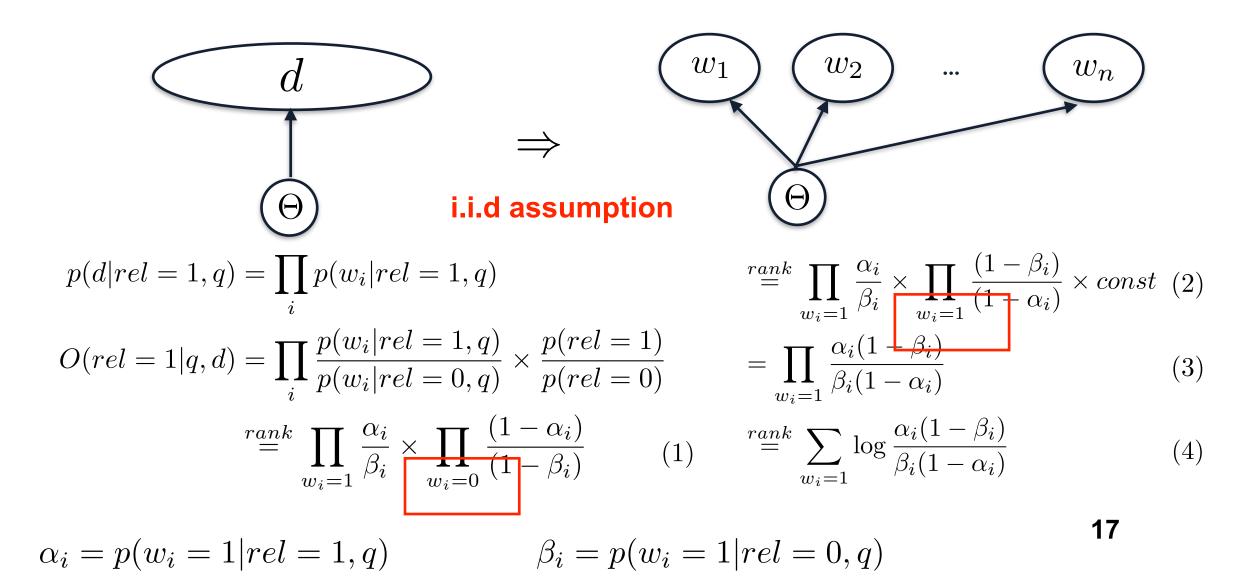
### **Estimating** p(rel = 1|q, d)



#### **Estimating** p(rel = 1|q, d)

$$p(rel = 1|q, d) \propto p(d|rel = 1, q)p(rel = 1) \qquad \qquad \begin{array}{l} \mbox{Problems with this estimation} \\ O(rel = 1|q, d) = \frac{p(rel = 1|q, d)}{p(ference|q|, d)} \\ \mbox{pifference}(q, d) = \frac{p(d|rel = 1, q)p(rel = 1)}{p(d|rel = 0, q)p(rel = 0)} \\ \mbox{odds} \\ \end{array}$$

#### Estimating the generative model p(d|rel = 1, q)



#### **RSJ model**

$$O(rel = 1|q, d) \stackrel{rank}{=} \sum_{w_i=1} \log \frac{\alpha_i (1 - \beta_i)}{\beta_i (1 - \alpha_i)}$$

(Robertson & Sparck Jones 76)

$$\alpha_i = p(w_i = 1 | q, rel = 1)$$
$$= \frac{count(w_i = 1, rel = 1) + 0.5}{count(rel = 1) + 1}$$

Probability for a word to appear in a relevant doc

$$\beta_i = p(w_i = 0 | q, rel = 0) \\= \frac{count(w_i = 0, rel = 0) + 0.5}{count(rel = 0) + 1}$$

Probability for a word to appear in a non-relevant doc

# **RSJ model: Summary**

- Uses only binary word occurrence (binary inference model), does not leverage TF information
  - RSJ model was designed for retrieving short text and abstract!
- Requires relevance judgment
  - No-relevance judgment version: [Croft & Harper 79]
- Performance is not as good as tuned vector-space model

#### How to improve RSJ based on these desiderata?

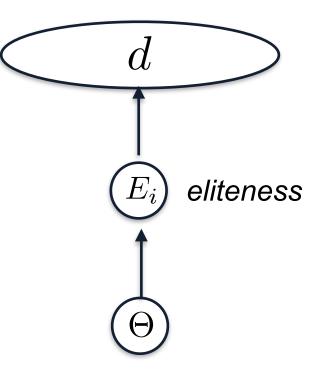
#### **Desiderata of retrieval models**

- Recall the desiderata of a retrieval models:
  - The importance of TF is sub-linear
  - Penalizing term with large document frequency using IDF
  - Pivot length normalization

#### How to improve RSJ based on these desiderata?

# Okapi/BM25

- Estimate probability using *eliteness* 
  - What is eliteness?
  - A term/word is elite if the document is about the concept denoted by the term
  - Eliteness is binary
  - Term occurrence depends on eliteness



#### Okapi/BM25

- Introduced in 1994
  - SOTA non-learning retrieval model

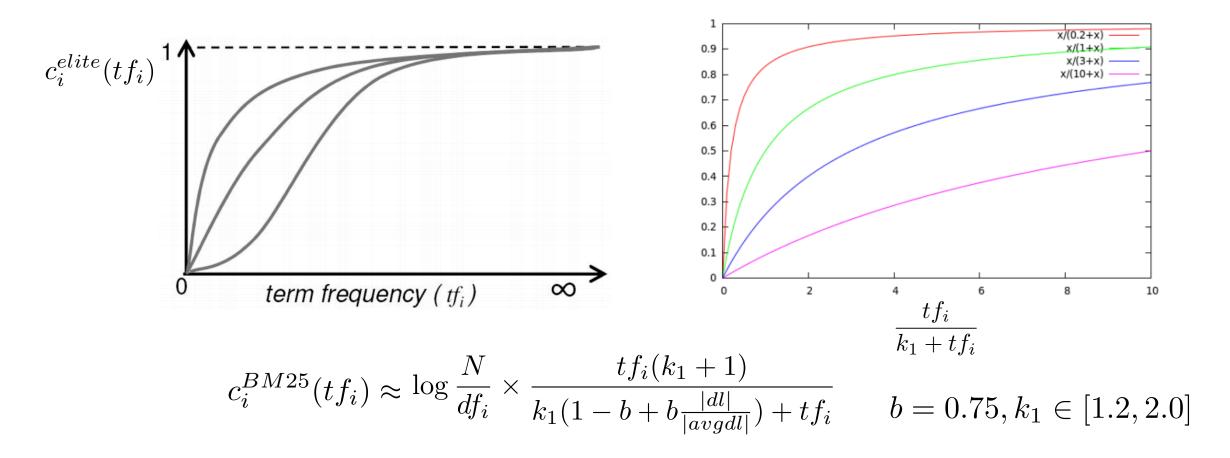
$$score(q, d) = \sum_{i \in q} c_i^{elite}(tf_i) tf_i = tf(i, d) \\ c_i^{elite}(tf_i) = \log \frac{p(w_i = tf_i | q, rel = 1)p(w_i = 0 | q, rel = 0)}{p(w_i = 0 | q, rel = 1)p(w_i = tf_i | q, rel = 0)} \\ p(w_i = tf_i | q, rel = 1) = p(w_i = tf_i | E_i = 1)p(E_i = 1 | q, rel) \\ + p(w_i = tf_i | E_i = 0)p(E_i = 0 | q, rel) \\ = \pi \frac{\lambda^{tf_i}}{tf_i!} e^{-\lambda} + (1 - \pi) \frac{\mu^{tf_i}}{tf_i!} e^{-\mu}$$
 (2 Poisson model)

#### Okapi/BM25

$$p(w_i = tf_i | q, rel = 1) = \pi \frac{\lambda^{tf_i}}{tf_i!} e^{-\lambda} + (1 - \pi) \frac{\mu^{tf_i}}{tf_i!} e^{-\mu}$$

- We do not know  $\,\lambda,\mu,\pi\,$
- Can we estimate  $\lambda, \mu, \pi$  ? Difficulty to estimate
- Designing a parameter-free model such that it simulates  $p(w_i = tf_i | q, rel = 1)$

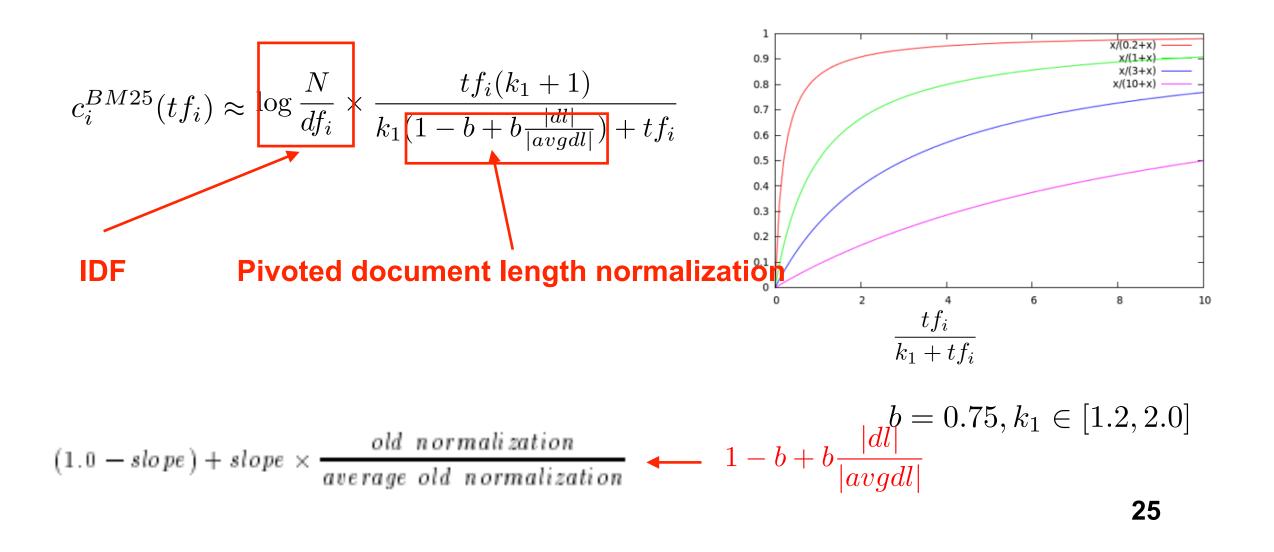
#### Simulating the 2-Poisson model



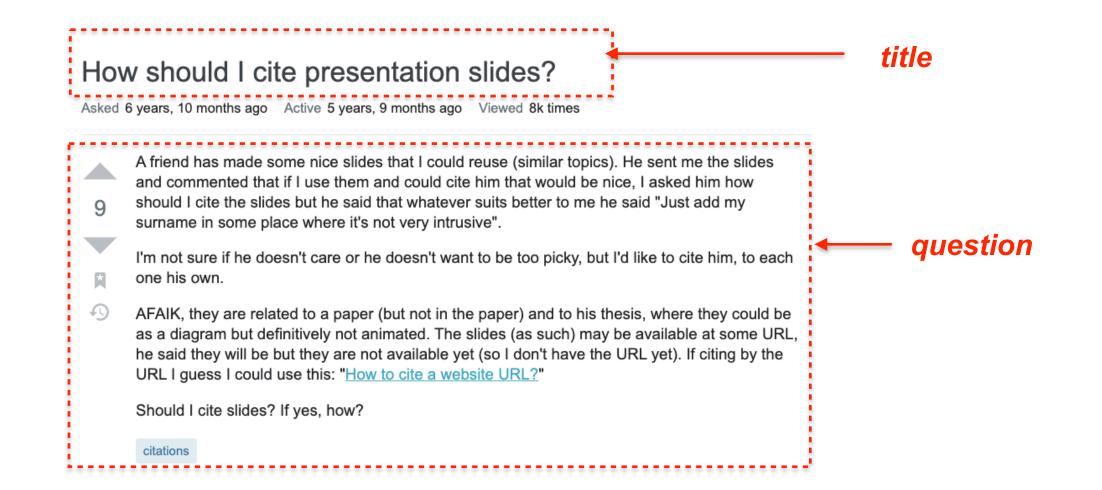
slides from Stanford CS276 Information retrieval

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#### **Analysis of BM25 formulation**



#### **Multi-field retrieval**



BM25F

$$score^{BM25F}(q,d) = \log \frac{N}{df_i} \times \frac{tf_i^F(k_1+1)}{k_1(1-b+b\frac{|dl|^F}{|avgdl|^F}) + tf_i^F}$$

• Each variable is estimated as the weighted sum of its field value

$$tf_i = \sum_{f} \alpha_f \times tf_{i,f} \qquad dl = \sum_{f} \alpha_f \times dl_f \qquad avgdl = \sum_{f} \alpha_f \times avgdl_f$$
  
parameter estimation using grid search

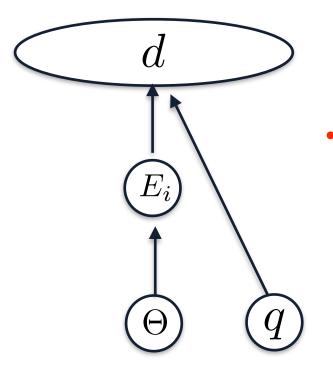
#### **Multi-field retrieval**

• BM25 outperforms TF-IDF in every field & combined

	[1.0, 0.0, 0.0]	[0.0, 1.0, 0.0]	[0.0, 0.0, 1.0]	[1.0, 0.5, 0.5]
Python, bm2, ndcg@10	0.319	0.322	0.293	0.378
Python, tfidf, ndcg@10	0.317	0.274	0.276	0.355
Java, bm2, ndcg@10	0.327	0.287	0.254	0.376
Java, tfidf, ndcg@10	0.315	0.258	0.238	0.349
Javascript, bm2, ndcg@10	0.349	0.330	0.267	0.407
Javascript, tfidf, ndcg@10	0.346	0.289	0.247	0.374

# Analysis on the n-Poisson model

• Advantage: BM25 is based on the 2-Poisson model



*eliteness*: *d* satisfies q's information need, when q is a single term

#### Disadvantages:

- For single term, documents will not fall cleanly into elite/non-elite set
- For multiple term, requires a combinatorial explosion of elite set
- Requires explicit indexing of the 'elite' words

#### Language model-based retrieval

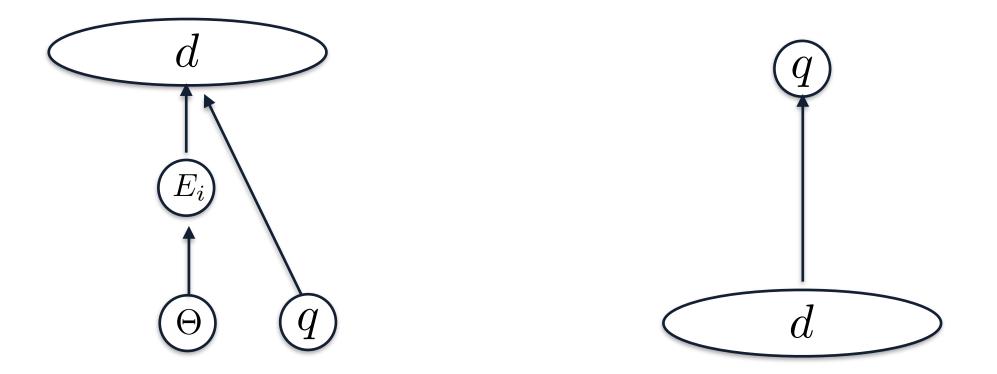
• A language model-based retrieval method [Ponte and Croft, 1998]

$$score(q,d) = \log p(q|d) = \prod_{i,w_i \in q} p(w_i = 1|d) \prod_{i,w_i \notin q} (1.0 - p(w_i = 1|d))$$

• Bernoulli -> multinomial

$$score(q,d) = \log p(q|d) = \prod_{w_i=1}^{V} p(w_i|d)^{c(w_i,q)} \qquad p(w_i|d) = \begin{cases} p_{seen}(w_i|d) & \text{if } w_i \text{ is seen in } d \\ \alpha_d p(w_i|C) & o.w. \end{cases}$$
$$rank \sum_{w_i=1}^{V} c(w_i,q) \log p(w_i|d)$$
$$corpus unigram LM$$

#### Language model-based retrieval



Disclaimer: the right figure is a schematic model, not a rigorous graphical model

#### Language model-based retrieval

$$\log p(q|d) = \sum_{w_i}^{V} c(w_i, q) \log p(w|d)$$

$$= \sum_{w_i, w_i \in d} c(w_i, q) \log p_{seen}(w_i|d) + \sum_{w_i, w_i \notin d} c(w_i, q) \log \alpha_d p(w_i|C)$$

$$\cdots$$

$$= \sum_{w_i, w_i \in d} c(w_i, q) \log \frac{p_{seen}(w_i|d)}{\alpha_d p(w_i|C)} + |q| \log \alpha_d + \sum_{w_i=1}^{V} c(w_i, q) \log p(w_i|C)$$

$$score^{LM}(q, d) \stackrel{rank}{=} \sum_{w_i, w_i \in d} c(w_i, q) \log \frac{p_{seen}(w_i|d)}{\alpha_d p(w_i|C)} + |q| \log \alpha_d \stackrel{constant}{\alpha_d p(w_i|C)}$$
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#### efficient to compute, general formulation

# Different senses of 'model' [Ponte and Croft, 98]

• First sense (high level): an abstraction of the retrieval task itself

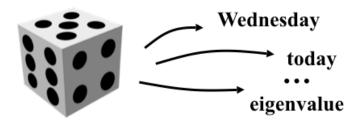


- Second sense (mid level): modeling the distribution, e.g., 2-Poisson model
- Thirds sense (low level): which statistical language model is used in  $p_{seen}(w_i|d)$

#### **Statistical language model**

- A probability distribution over word sequences
  - p("Today is Wednesday") ≈ 0.001

  - p("The eigenvalue is positive") ≈ 0.00001
- Unigram language model
  - Generate text by generating each word INDEPENDENTLY
  - Thus,  $p(w_1 w_2 ... w_n) = p(w_1)p(w_2)...p(w_n)$
  - Parameters: {p( $t_i$ )} p( $t_1$ )+...+p( $t_N$ )=1 (N is voc. size)



p("today is Wed")= p("today")p("is")p("Wed")=  $0.0002 \times 0.001 \times 0.000015$ 

### Notes on language model-based retrieval

- Advantages:
  - Avoided the disadvantages in eliteness
  - Defines a general framework, more accurate  $p_{seen}(w_i|d)$  can further improve the model
  - In some cases, has outperformed BM25

#### • Disadvantages:

- The assumed equivalence between query and document is unrealistic
- Only studied unigram language model
- Performance is not always good

#### Equivalence to KL-divergence retrieval model

$$score^{LM}(q,d) \stackrel{rank}{=} \sum_{w_i,w_i \in d} c(w_i,q) \log \frac{p_{seen}(w_i|d)}{\alpha_d p(w_i|C)} + |q| \log \alpha_d$$

• KL divergence

why

$$D(p||q) = \sum_{x} p(x) \log \frac{p(x)}{q(x)}$$

$$-D(\hat{\theta}_{q}||\hat{\theta}_{d}) = \sum_{w_{i}=1}^{V} p(w_{i}|\hat{\theta}_{q}) \log p(w_{i}|\hat{\theta}_{q}) + \left(-\sum_{w_{i}=1}^{V} p(w_{i}|\hat{\theta}_{q}) \log p(w_{i}|\hat{\theta}_{d})\right)$$

$$\therefore \qquad \text{smoothed} \qquad \text{constant}$$
not the opposite?
$$= \sum_{w_{i},w_{i} \in d} p(w_{i}|\hat{\theta}_{q}) \log \frac{p_{seen}(w_{i}|d)}{\alpha_{d}p(w_{i}|C)} + \log \alpha_{d} \qquad \text{(Eq. 1)}$$

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Notes on the KL-divergence retrieval formula and Dirichlet prior smoothing

#### **Estimating** $p_{seen}(w_i|d)$

• Estimating  $p_{seen}(w_i|d)$  based on the maximum likelihood estimation

$$p_{seen}(w_i|d) = \frac{count(w_i)}{|dl|}$$

- Disadvantage: if the word is unseen, probability will be 0
- Solution: language model smoothing:

$$p_s(w_i|d) = \frac{c(w_i, d) + \mu p(w_i|C)}{|d| + \mu} \qquad \alpha_d = \frac{\mu}{\mu + |d|} \quad \text{(plug in Eq. 1)}$$
$$= \frac{|d|}{|d| + \mu} p(w_i d) + \frac{\mu}{|d| + \mu} p(w_i|C) \qquad \text{Dirichlet smoothing}$$

#### **Estimating** $p_{seen}(w_i|d)$

• Dirichlet smoothing

$$score^{Dir}(q,d) = \sum_{w_i, w_i \in d, p(w_i|\hat{\theta}_q)} p(w_i|\hat{\theta}_q) \log(1 + \frac{count(w_i,d)}{\mu p(w_i|C)}) + \log\frac{\mu}{\mu + |dl|}$$

• Jelinek-Mercer smoothing

$$score^{JM}(q,d) = \sum_{w_i, w_i \in d, p(w_i|\hat{\theta}_q)} p(w_i|\hat{\theta}_q) \log\left(1 + \frac{(1-\lambda)count(w_i,d)}{\lambda p(w_i|C)}\right)$$

# Other smoothing methods

- Additive smoothing
- Good-Turing smoothing
- Absolute discounting
- Kneser-ney smoothing

## Tuning parameters in smoothing models [Zhai and Lafferty 02]

$$score^{Dir}(q,d) = \sum_{w_i, w_i \in d, p(w_i|\hat{\theta}_q)} p(w_i|\hat{\theta}_q) \log(1 + \frac{count(w_i,d)}{\mu p(w_i|C)}) + \log\frac{\mu}{\mu + |dl|}$$

- Tuning parameter  $\mu$  using "leave-one-out" method

$$\hat{\mu} = argmax_{\mu} \sum_{w_i=1}^{V} \sum_{d} \log p(w_i|d; w_i \notin d)$$
 remove  $w_i$ 

• Estimating parameter using Newton's method (2nd derivative)

#### Tuning parameters in smoothing models [Zhai and Lafferty 02]

$$score^{JM}(q,d) = \sum_{w_i, w_i \in d, p(w_i|\hat{\theta}_q)} p(w_i|\hat{\theta}_q) \log\left(1 + \frac{(1-\lambda)count(w_i,d)}{\lambda p(w_i|C)}\right)$$

- Tuning parameter  $\boldsymbol{\lambda}$  using MLE for the query probability

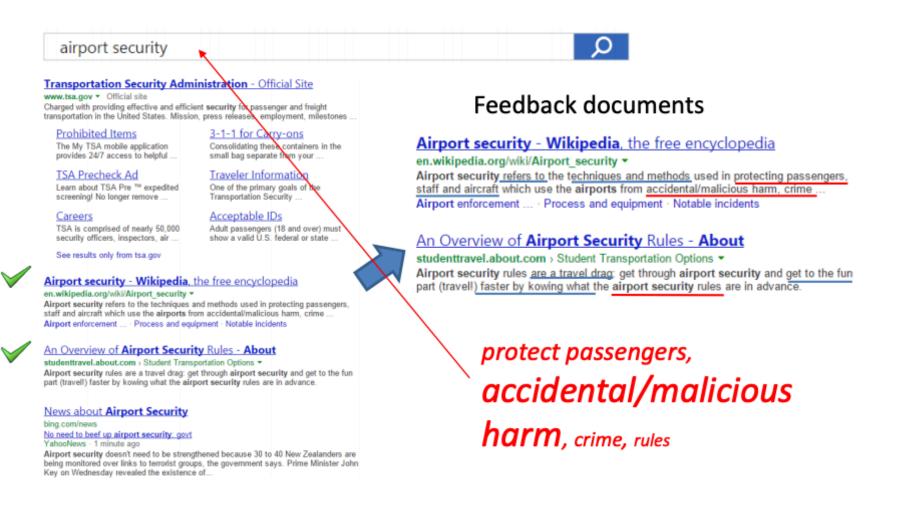
$$p(q|\lambda, C) = \sum_{d} \pi_{d} \prod_{w_{i} \in q} ((1-\lambda)p(w_{i}|d) + \lambda p(w_{i}|C))$$

• EM algorithm:

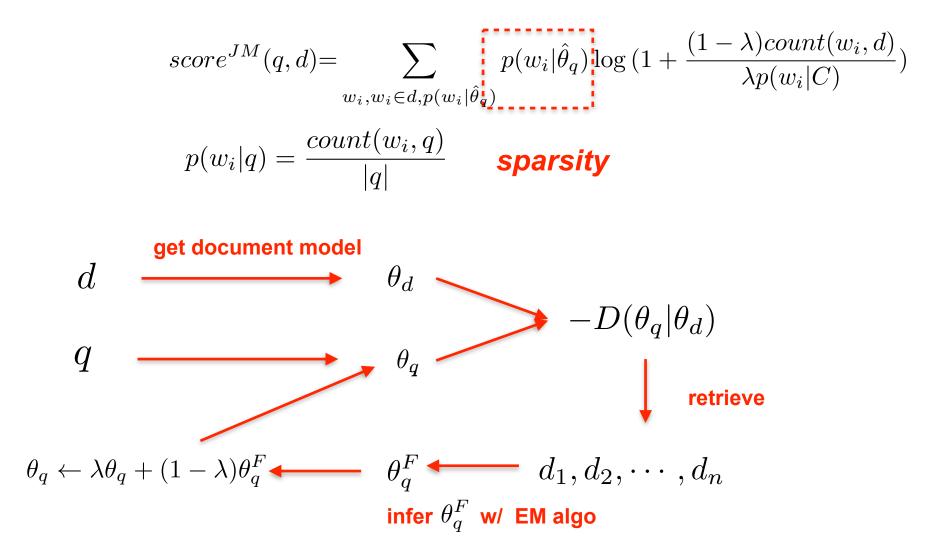
$$\pi_{d}^{(k+1)} = \frac{\pi_{d}^{(k)} \prod_{w_{i} \in q} \left( \left( 1 - \lambda^{(k)} \right) p\left( w_{i} \mid d \right) + \lambda^{(k)} p\left( w_{i} \mid C \right) \right)}{\sum_{d} \pi_{d}^{(k)} \prod_{w_{i} \in q} \left( \left( 1 - \lambda^{(k)} \right) p\left( w_{i} \mid d \right) + \lambda^{(k)} p\left( w_{i} \mid C \right) \right)}$$
$$\lambda^{(k+1)} = \frac{1}{|q|} \sum_{d} \pi_{d}^{(k+1)} \sum_{w_{i} \in q} \frac{\lambda^{(k)} p\left( w_{i} \mid C \right)}{\left( 1 - \lambda^{(k)} \right) p\left( w_{i} \mid d \right) + \lambda^{(k)} p\left( w_{i} \mid C \right)}$$

Two-stage language models for information retrieval

# Feedback language model [Zhai and Lafferty 01]



#### Feedback language model [Zhai and Lafferty 01]



Model-based feedback in the language modeling approach to information retrieval

# Evaluation on smoothing methods [Zhai & Lafferty 02]

Collection	query	Optimal-JM	<b>Optimal-Dir</b>	Auto-2stage
	SK	20.3%	23.0%	22.2%*
	LK	36.8%	37.6%	37.4%
	SV	18.8%	20.9%	20.4%
AP88-89	LV	28.8%	<b>29.8%</b>	29.2%
	SK	19.4%	22.3%	21.8%*
	LK	34.8%	35.3%	35.8%
	SV	17.2%	<b>19.6%</b>	19.9%
WSJ87-92	LV	27.7%	28.2%	28.8%*
	SK	17.9%	21.5%	20.0%
	LK	32.6%	32.6%	32.2%
	SV	15.6%	18.5%	18.1%
ZIFF1-2	LV	26.7%	27.9%	27.9%*

# **Evaluation on smoothing methods [Zhai & Lafferty 01b]**

collec	tion	Simple LM	Mixture	Improv.	Div.Min.	Improv.
	AvgPr	0.21	0.296	+41%	0.295	+40%
	InitPr	0.617	0.591	-4%	0.617	+0%
AP88-89	Recall	3067/4805	3888/4805	+27%	3665/4805	+19%
	AvgPr	0.256	0.282	+10%	0.269	+5%
	InitPr	0.729	0.707	-3%	0.705	-3%
TREC8	Recall	2853/4728	3160/4728	+11%	3129/4728	+10%
	AvgPr	0.281	0.306	+9%	0.312	+11%
	InitPr	0.742	0.732	-1%	0.728	-2%
WEB	Recall	1755/2279	1758/2279	+0%	1798/2279	+2%

#### Comparison between BM25 and LM [Bennett et al. 2008]

Collection	Method	Parameter	MAP	R-Prec.	Prec@10
Trec8 T	Okapi BM25	Okapi	0.2292	0.2820	0.4380
	JM	$\lambda = 0.7$	0.2310 (p=0.8181)	0.2889 (p=0.3495)	0.4220 (p=0.3824)
	Dir	$\mu = 2,000$	<b>0.2470</b> (p=0.0757)	0.2911 (p=0.3739)	<b>0.4560</b> (p=0.3710)
	Dis	$\delta = 0.7$	0.2384 (p=0.0686)	0.2935 (p=0.0776)	0.4440 (p=0.6727)
	Two-Stage	auto	0.2406 (p=0.0650)	<b>0.2953</b> (p=0.0369)	0.4260 (p=0.4282)
Trec8 TD	Okapi BM25	Okapi	0.2528	0.2908	0.4640
	JM	$\lambda = 0.7$	$0.2582 \ (p=0.5226)$	0.3038 (p=0.1886)	0.4600 (p=0.8372)
	Dir	$\mu = 2,000$	<b>0.2621</b> (p=0.3308)	0.3043 (p=0.1587)	0.4460 (p=0.3034)
	Dis	$\delta = 0.7$	0.2599 (p=0.1737)	0.3105 (p=0.0203)	<b>0.4880</b> (p=0.1534)
	Two-Stage	auto	0.2445 (p=0.2455)	0.2933 (p=0.7698)	0.4400 (p=0.1351)

However, BM25 outperforms LM in other cases

## Summary on parameter tuning

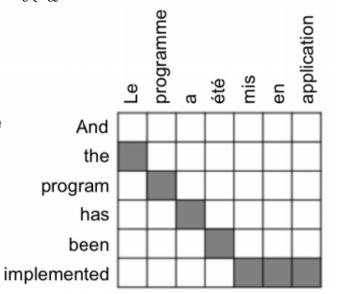
- RSJ: no parameter
- BM25: Due to the formulation of two-Poisson, parameters are difficult to estimate, so use a parameter free version to replace it
- Language model
  - Leave-one-out
  - EM algorithm

# Translation-based language model [Xue et al. 2008]

The retrieval model can benefit from incorporating knowledge in the formulation

$$p_{l}w_{i}|d) = \frac{|dl|}{|dl| + \lambda} p_{mix}(w_{i}|d) + \frac{\lambda}{|dl| + \lambda} p(w_{i}|C)$$
$$p_{mix}(w_{i}|d) = (1 - \beta)p(w_{i}|d) + \beta \sum_{t \in d} p_{tr}(w_{i}|t)p(t|d)$$

• Translation matrix:



Retrieval Models for Question and Answer Archives

#### Performance of translation based LM [Xue et al. 2008]

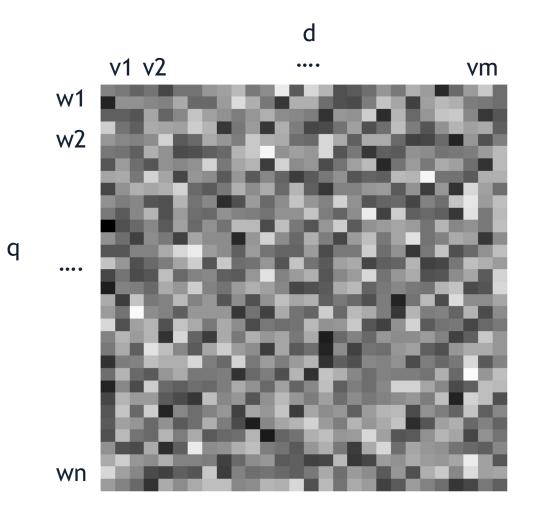
	Python		Java			JavaScript			
	R	@5	@10	R	@5	@10	R	@5	@10
TF-IDF	.299	.301	.360	.285	.282	.352	.305	.315	.378
BM25	.313	.320	.384	.311	.321	.382	.329	.344	.412
TransLM	.468	.502	.553	.455	.487	.544	.483	.528	.573

	P		J F	
			Wondir	
Type	Model	Trans Prob	MAP	P@10
Type I	LM		0.3217	0.2211
	Okapi		0.3207	0.2158
	RM		0.3401	0.2395
Type II	LM-Comb		0.3791	0.2368
Type III	Murdock	P(Q A)	0.3566	0.25
	Murdock	P(A Q)	0.3658	0.2526
	Jeon	P(Q A)	0.3546	0.25
	Jeon	P(A Q)	0.3658	0.2526
	TransLM	P(Q A)	0.379	0.2658
	TransLM	P(A Q)	0.4059	0.2684

## **Discussion on query length**

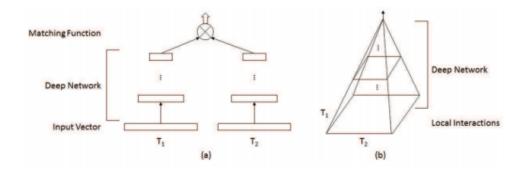
- What if the query is very long?
  - For example, the query is a paragraph or a document
  - The problem of retrieval is turned into a matching problem
    - i.e., semantic matching

## Deep semantic matching [Pang et al. 2016]



each cell:

distributed representation of words (word2vec)



## **Question asking protocol**

- Regrading requests: email TA, cc myself, titled [CS589 regrading]
- Deadline extension requests: email myself, titled [CS589 deadline]
- Dropping: email myself, titled [CS589 drop]
- All technical questions: Piazza
  - Homework description clarification
  - Clarification on course materials
- Having trouble with homework: join my office hour directly, no need to email me
  - If you have a time conflict, email me & schedule another time
- Project discussion: join my office hour
- Ask any common questions shared by the class on Piazza

#### Homework 1

- Homework 1 is released in Canvas:
- Implementing TF-IDF and BM25 on the LinkSO dataset:
  - <u>https://sit.instructure.com/courses/44342/assignments/218604</u>