CS 589 Fall 2020

Maximum likelihood estimation

Expectation maximization

Instructor: Susan Liu
TA: Huihui Liu

Stevens Institute of Technology

Recap of Lecture 2

- RSJ: no parameter
- BM25: Due to the formulation of two-Poisson, parameters are difficult to estimate, so use a parameter free version to replace it
- Language model based retrieval model
  - Leave-one-out
  - EM algorithm

Maximum likelihood estimation

- RSJ:

PRP: rank documents by $p(rel = 1|q, d)$

$$p(rel = 1|q, d) \propto p(d|rel = 1, q)p(rel = 1)$$

$$\alpha_i = p(w_i = 1|q, rel = 1) = \frac{count(w_i = 1, rel = 1) + 0.5}{count(rel = 1) + 1}$$

$$\beta_i = p(w_i = 0|q, rel = 0) = \frac{count(w_i = 0, rel = 0) + 0.5}{count(rel = 0) + 1}$$

- Language model

$$\hat{\mu} = argmax_{\mu} \sum_{w_i=1}^{V} \sum_{d} \log p(w_i|d; w_i \notin d)$$

Today's lecture

- Maximum likelihood estimation
- Expectation maximization
  - Coin-topic problem
  - Using EM algorithm to remove stop words
- Mixture of topic models
  - Probabilistic latent semantic analysis
  - PLSA with partial labels

Maximum likelihood estimation

$$max_\theta \sum_i \log P(\mathbf{x}_i;\theta)$$

$\mathbf{x}_i$	**observations**	e.g., mice weights
$P(\mathbf{x}_i;\theta)$	**likelihood**	$e.g., \mathcal{N}(x_i;\mu,\sigma^2)$
θ	**parameters**	

If the optimal solution is within θ's space $\mathcal{S}$: $\frac{\partial \sum_i \log P(\mathbf{x}_i;\theta)}{\partial \theta} = 0$ at $\theta = \hat{\theta}_{ML}$

Expectation maximization algorithm

- How to estimate the optimal θ?

- Expectation maximization (EM) algorithm:
  - Relies on the concept of ***complete data*** space
  - Iterative and alternative between conditional expectation and maximization steps

l(theta): Incomplete data space: observation, e.g., documents $p(\mathbf{x};\theta)$

l_{cd}(theta): Complete data space: observation + latent variables, e.g., topic $p(\mathbf{x},z|\theta)$

Expectation maximization algorithm

- Estimating the incomplete probability using the complete space

$$p(\mathbf{x};\theta) = \sum_{k=1}^{K} p(\mathbf{x}|z;\theta)p(z=k)$$ **discrete space**

$$p(\mathbf{x};\theta) = \int_z p(\mathbf{x}|z;\theta)dp(z)$$ **continuous space**

- EM algorithm: repeat n=1…N:

E step: $$Q(\theta|\hat{\theta}^{(n)}) = \mathbb{E}_{\hat{\theta}^{(n)}}[\log p(\mathbf{x}, z|\theta)]$$

M step: $$\hat{\theta}^{(n+1)} = arg \max_{\theta \in \mathcal{S}} Q(\theta|\hat{\theta}^{(n)})$$

Expectation maximization: convergence guarantee

- **Theorem**: the likelihood of observation, $\log p(\mathbf{x}; \theta^{(n)})$, monotonously increases with *n*

$$p(z, \mathbf{x}|\theta) = p(z|\mathbf{x}, \theta)p(\mathbf{x}|\theta)$$

$$\log p(z, \mathbf{x}|\theta) = \log p(z|\mathbf{x}, \theta) + \log p(\mathbf{x}|\theta)$$

$$l(\theta) = l_{cd}(\theta) - \log p(z|\mathbf{x}; \theta)$$

$$l(\theta^{(n+1)}) - l(\theta^{(n)}) = l_{cd}(\theta^{(n+1)}) - l_{cd}(\theta^{(n)}) + \log\left[p(z|\mathbf{x}, \theta^{(n)})/p(z|\mathbf{x}, \theta^{(n+1)})\right]$$

Expectation maximization: convergence guarantee

- Take the expectation over $p(z|\mathbf{x},\theta^{(n)})$ on both side

$$l(\theta^{(n+1)}) - l(\theta^{(n)}) = l_{cd}(\theta^{(n+1)}) - l_{cd}(\theta^{(n)}) + \log\left[p(z|\mathbf{x},\theta^{(n)})/p(z|\mathbf{x},\theta^{(n+1)})\right]$$

$$\Rightarrow l(\theta^{(n+1)}) - l(\theta^{(n)}) = \boxed{\mathbb{E}_{p(z|\mathbf{x},\theta^{(n)})}[l_{cd}(\theta^{(n+1)})] - \mathbb{E}_{p(z|\mathbf{x},\theta^{(n)})}[l_{cd}(\theta^{(n)})]} + D_{KL}(p(z|\mathbf{x},\theta^{(n+1)}\|p(z|\mathbf{x},\theta^{(n)}))$$

$$Q(\theta^{(n+1)}|\hat{\theta}^{(n)}) - Q(\theta^{(n)}|\hat{\theta}^{(n)})$$

EM chooses $\theta^{(n+1)}$ to maximize $Q(\theta^{(n+1)}|\hat{\theta}^{(n)})$ KL divergence always nonneg

$$\Rightarrow l(\theta^{(n+1)}) \geq l(\theta^{(n)})$$

An example problem: Coin-topic problem

- Author H and author T are co-authoring a paper in the following way:
  - At each time, they toss a coin to write the next word. If it’s “head”, author H writes the next word, if it’s “tail”, author T writes the next word. The probability for “head” is λ
  - The head author selects the next word by randomly sampling from $p(w|H)$, so does the tail author
- **Problem**: estimating the parameters that maximizes the document likelihood

Coin-topic problem: known p(v|T), unknown λ

- Maximum likelihood estimation:

$$\max_{\lambda} \sum_{i} \sum_{v=1}^{V} \log(\lambda p(w_i = v \mid H) + (1-\lambda) p(w_i = v \mid T))$$

- Suppose both head and tail distributions are known, e.g.:

	the	computer	data	baseball	game	interesting
p(w|T)	0.2	0.05	0.05	0.4	0.2	0.1
p(w|H)	0.25	0.2	0.2	0.15	0.1	0.1

Expectation maximization

- We use $p(Z|v)$ to represent the hidden variable, i.e., whether the topic for word v is head or tail topic

$$\log p(d \mid \lambda) = \sum_i \sum_{v=1}^{V} (p(Z=0 \mid w_i = v) \log \lambda p(w_i = v \mid H) + (1 - p(Z=0 \mid w_i = v)) \log(1-\lambda) p(w_i = v \mid T))$$

- Take the derivative of $\log p(d \mid \lambda)$ over lambda:

$$\sum_i \sum_{v=1}^{V} (p(Z=0 \mid w_i = v) \frac{1}{\lambda} + (1 - p(Z=0 \mid w_i = v)) \frac{1}{1-\lambda}) = 0$$

$$\Rightarrow \lambda^{(n+1)} = \frac{1}{|d|} \sum_{v=1}^{V} count(d, v) p^{(n)}(Z=0 \mid v)$$ **(M step)**

Expectation maximization

- We use $p(Z|v)$ to represent the hidden variable, i.e., whether the topic for word v is head or tail topic

$$\log p(d \mid \lambda) = \sum_i \sum_{v=1}^{V} (p(Z=0 \mid w_i = v) \log \lambda p(w_i = v \mid H) + (1 - p(Z=0 \mid w_i = v)) \log(1-\lambda) p(w_i = v \mid T))$$

- E step: the standard derivation is to apply Bayes theorem:

$$p^{(n+1)}(Z=0 \mid v; d) \propto p(v \mid Z=0) p(Z=0) = p(v \mid T) \lambda^{(n)}$$
$$p^{(n+1)}(Z=1 \mid v; d) \propto p(v \mid Z=1) p(Z=1) = p(v \mid H) \left(1 - \lambda^{(n)}\right)$$

Coin-topic problem: unknown topic, known λ

- For the same coin topic problem, assume lambda is known whereas $p(w|H)$ is unknown, estimate $p(w|H)$

$$\underset{p(w|H)}{\cancel{\underset{\lambda}{\max}}} \sum_i \sum_{v=1}^{V} \left(p\left(Z=0 \mid w_i=v\right) \log \lambda p\left(w_i=v \mid H\right) + \left(1-p\left(Z=0 \mid w_i=v\right)\right) \log (1-\lambda) p\left(w_i=v \mid T\right)\right)$$

$$-\eta\left(\sum_v p(v \mid H)-1\right)$$

- Take the derivative and set to 0, we can get (M step):

$$p(v \mid H) \propto \sum_i \sum_v 1\left[w_i==v\right] p(Z=0 \mid v)$$

$$\Rightarrow p^{(n+1)}(v \mid H)=\frac{\sum_i 1\left[w_i=v\right] \cdot p^{(n)}(Z=0 \mid v)}{\sum_u \sum_i 1\left[w_i==u\right] \cdot p^{(n)}(Z=0 \mid u)}$$

- E step follows the same posterior estimation as the previous slide

Coin-topic problem: unknown topic, known λ

- For the same coin topic problem, assume lambda is known whereas $p(w|H)$ is unknown, estimate $p(w|H)$

- Application: removing background topic
    - Suppose p(w|H) is the main topic (computer game)
    - p(w|T) is the background topic: the: 0.3, a: 0.2, …,
    - The mixture of head and tail topic is dominated by background words:
  - After stop words removal, the true topic p(w|T) is “revealed”:

Coin-topic problem: unknown topic and λ

- Suppose both p(w|H) and lambda are unknown:

$$\Rightarrow \lambda^{(n+1)} = \frac{1}{|d|} \sum_{v=1}^{V} count(d, v) p^{(n)}(Z = 0 \mid v)$$

(M step of unknown lambda)

$$p(v \mid H) \propto \sum_i \sum_v 1\left[w_i == v\right] p(Z = 0 \mid v)$$

$$\Rightarrow p^{(n+1)}(v \mid H) = \frac{\sum_i 1\left[w_i = v\right] \cdot p^{(n)}(Z = 0 \mid v)}{\sum_u \sum_i 1\left[w_i == u\right] \cdot p^{(n)}(Z = 0 \mid u)}$$

(M step of unknown topic)

$$p^{(n+1)}(Z = 0 \mid v; d) \propto p(v \mid Z = 0) p(Z = 0) = p(v \mid T) \lambda^{(n)}$$
$$p^{(n+1)}(Z = 1 \mid v; d) \propto p(v \mid Z = 1) p(Z = 1) = p(v \mid H) \left(1 - \lambda^{(n)}\right)$$

(E step)

Applications of Coin Topic Problem for Text Mining

- Application Scenarios:
  - p(w|H) & p(w|T) are known; estimate λ
  - p(w|H) & λ are known; estimate p(w|T)
  - p(w|H) is known; estimate λ & p(w|T)
  - λ is known; estimate p(w|H)& p(w|T)
  - Estimate λ, p(w|H), p(w|T)

how much percent of the document is about computer game?

30% of the doc is about computer game, what's the other topic about?

The doc is about computer game, is it also about some other topic, and if so to what extent?

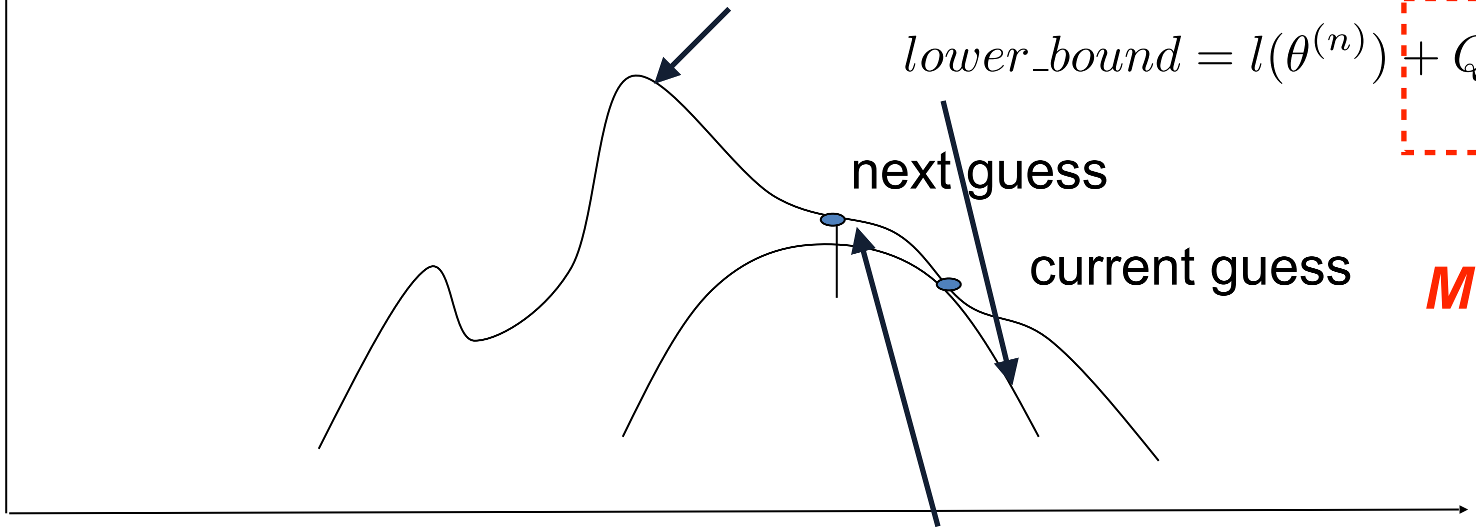
30% of the doc is about one topic and 70% is about another, what are these two topics?

The doc is about two subtopics, find out what these two subtopics are and to what extent the doc covers each.

Expectation maximization as hill climbing

$$l(\theta) = l(\theta^{(n)}) + Q(\theta; \hat{\theta}^{(n)}) - Q(\theta^{(n)}; \hat{\theta}^{(n)}) + D_{KL}(p(z|\mathbf{x}, \hat{\theta}^{n} \| p(z|\mathbf{x}, \hat{\theta}))$$

$$lower\_bound = l(\theta^{(n)}) \boxed{+ Q(\theta; \hat{\theta}^{(n)})} - Q(\theta^{(n)}; \hat{\theta}^{(n)})$$



M step object

converge to local optimal

Slides from UIUC CS510

EM algorithm in action

- Log likelihood increases:

Word	#	$p(w\|\theta_B)$	Iteration 1		Iteration 2		Iteration 3	
			$P(w\|\theta)$	$p(z=0\|w)$	$P(w\|\theta)$	$P(z=0\|w)$	$P(w\|\theta)$	$P(z=0\|w)$
The	4	0.5	**0.25**	0.33	**0.20**	0.29	**0.18**	0.26
Paper	2	0.3	**0.25**	0.45	**0.14**	0.32	**0.10**	0.25
Text	4	0.1	**0.25**	0.71	**0.44**	0.81	**0.50**	0.93
Mining	2	0.1	**0.25**	0.71	**0.22**	0.69	**0.22**	0.69
Log-Likelihood			-16.96		-16.13		-16.02	

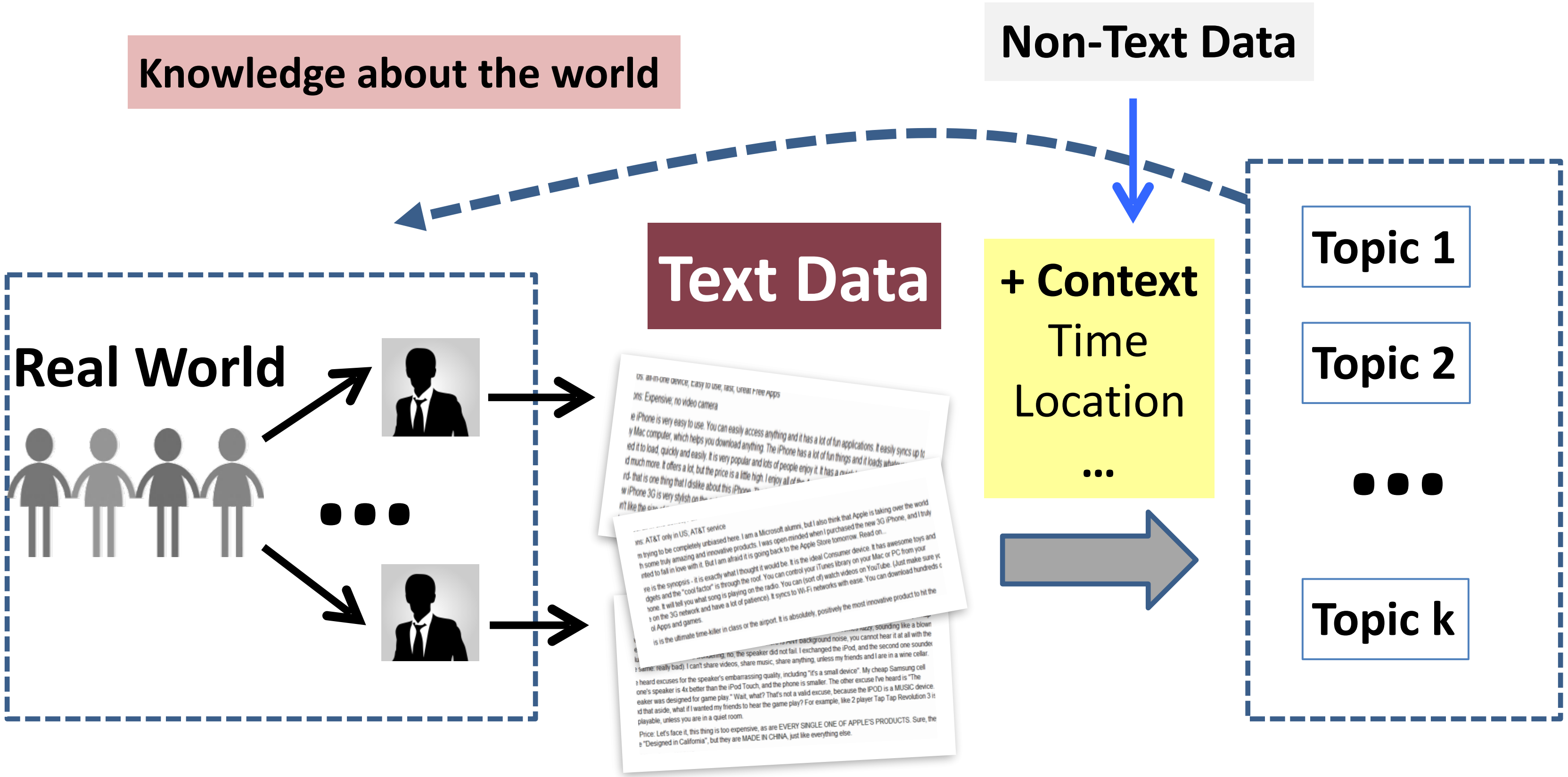
Topic models and analysis

- Topic ≈ main idea discussed in text data
  - Theme/subject of a discussion or conversation
  - Different granularities (e.g., topic of a sentence, an article, etc.)

- Many applications require discovery of topics in text
  - What are Twitter users talking about today?
  - What are the current research topics in data mining? How are they different from those 5 years ago?
  - What do people like about the iPhone 6? What do they dislike?
  - What were the major topics debated in 2012 presidential election?

Slides from UIUC CS510

Lifecycle of topic and text data

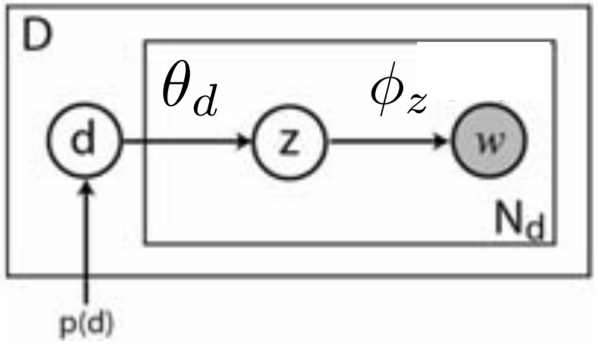


Slides from UIUC CS510

A generative process of documents

- Assume documents are generated by sampling words from k latent topics

- For each document d:
  - For each token position i
  - Choose a topic z ~ Multinomial(θ_d)
  - Choose a term w ~ Multinomial(ϕ_z)

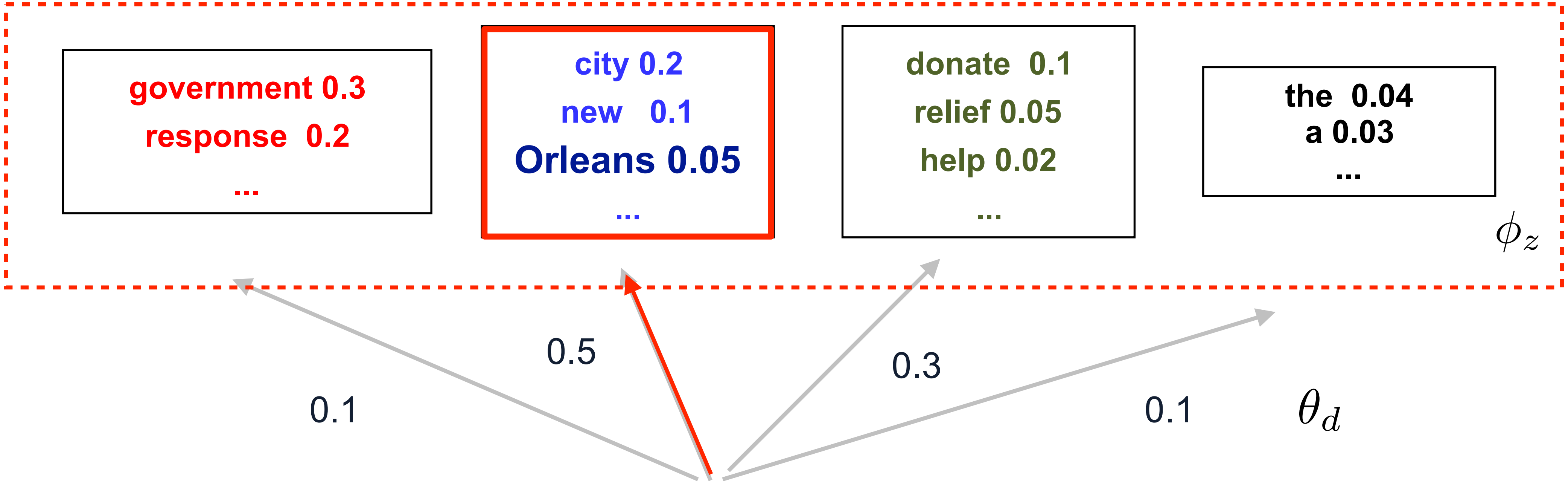


Probabilistic Latent Semantic Analysis. Thomas Hoffman. 2001.

Review of PLSA

Phi T x V, V: vocabulary size ~50,000, T: #topics, T=20

theta D x T, D: #documents: ~10,000, T: #topics, T=20



[Criticism of government response to the hurricane ..] to the [flooding of New Orleans

Step 1: sample a topic: topic 2

Step 2: sample a word from topic 2

Probabilistic Latent Semantic Analysis. Thomas Hoffman. 2001.

Document as a Sample of Mixed Topics

Topic θ_1

government 0.3
response 0.2
...

Topic θ_2

city 0.2
new 0.1
orleans 0.05
...

. . .

Topic θ_k

donate 0.1
relief 0.05
help 0.02
...

Background θ_B

the 0.04
a 0.03
...

Blog article about "Hurricane Katrina"

[Criticism of government response to the hurricane primarily consisted of criticism of its response to the approach of the storm and its aftermath, specifically in the delayed response] to the [flooding of New Orleans. … 80% of the 1.3 million residents of the greater New Orleans metropolitan area evacuated] …[Over seventy countries pledged monetary donations or other assistance]. …

proportion of topics

Probabilistic latent semantic analysis

$$p(d_i = w|\Phi, \theta_d) = \sum_{z=1}^{T} \phi_{z,w}\theta_{d,z}$$

$$p(\mathcal{W}|\Phi, \Theta)$$

$$= \prod_{d=1}^{D} \prod_{d_i=1}^{N_d} \sum_{z=1}^{T} \phi_{z,w}\theta_{d,z}$$

$$= \prod_{d=1}^{D} \prod_{w=1}^{V} (\sum_{z=1}^{T} \phi_{z,w}\theta_{d,z})^{count(d,w)}$$

$$arg \max_{\Phi,\Theta} [\log p(\mathcal{W}|\Phi, \Theta) + \sum_{d=1}^{D} \lambda_d (1 - \sum_{z=1}^{T} \theta_{(d,z)}) + \sum_{z=1}^{T} \sigma_k (1 - \sum_{w=1}^{V} \phi_{z,w})]$$

Probabilistic latent semantic analysis

- Use $R_{(w_{d_i},z)}$ to represent which topic d_i in document d comes from (repeated same tokens are from the same topic)

$$\mathcal{L} = \log p(\mathcal{W} \mid \mathbf{R}, \Phi, \Theta) = \sum_{d}^{D} \sum_{d_i}^{N_d} \sum_{z}^{T} R_{\left(w_{d_i} z\right)} \left(\log \phi_{(z,w_{di})} + \log \theta_{(d,z)}\right) \cdot \mathbb{1}\left[z_{d,i} == z\right]$$

$$+ \left(\sum_{d=1}^{D} \lambda_d \left(1 - \sum_{z=1}^{T} \theta_{(d,z)}\right) + \sum_{z=1}^{T} \sigma_k \left(1 - \sum_{w=1}^{V} \phi_{z,w}\right)\right]$$

- **M step: set the derivative of L to 0:**

$$\theta_{d,z} \propto \sum_{v=1}^{V} R_{d,v,z} count(v,d)$$

$$\phi_{z,v} \propto \sum_{d=1}^{D} \mathbb{1}\left[z_{d,v} == z\right] count(v,d)$$

E step:

$$R_{(w_{d_i},z)} \propto \phi_{z,v} \theta_{d,z}$$

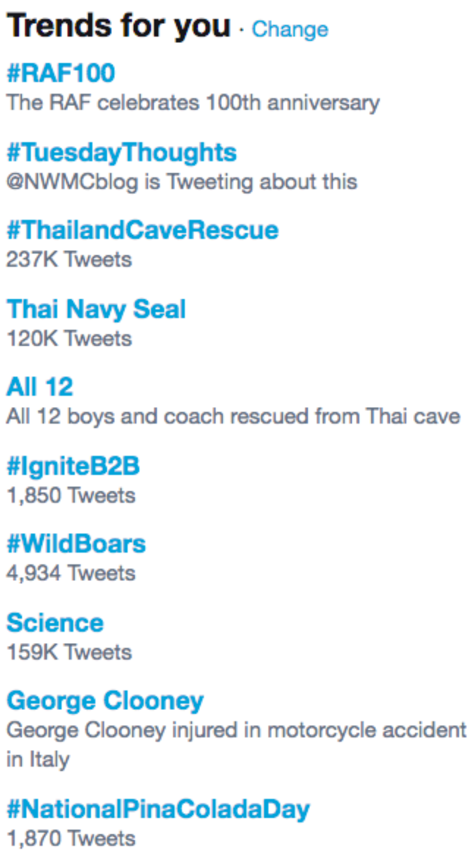
Probabilistic latent semantic analysis: partially available labels

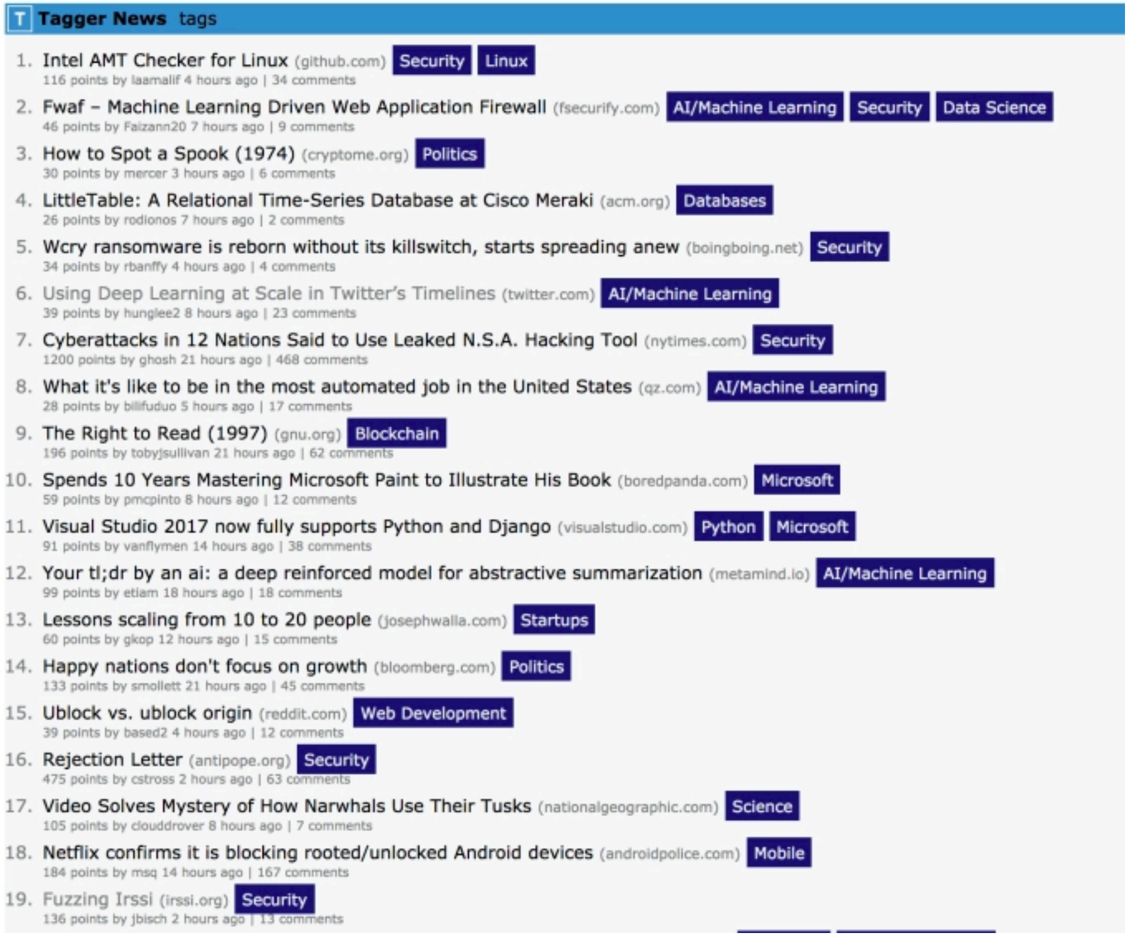
- Generalized topic modeling:
  - Each document can contain just one topic, e.g., short documents
  - That is, topic inference = topic classification

- If we already know the document tags for a part of the documents, does the partial labels help us make better predictions for the entire corpus? **(Homework 3)**

- Example: news tagging, StackOverflow question tagging

Probabilistic latent semantic analysis: partially available labels





Homework 3

- Suppose each document has only 1 topic. We have two document set: S1 (100 documents) contains all the tagged documents; S2 (10,000 documents) contains all the untagged documents. Each tag is a topic, there are only 2 topics

- (Part 1): derive the EM algorithm using pLSA that maximizes the probability of the observed document, given the known topics from S1

- (Part 2): implement your EM algorithm, output the predicted topic for each document in S2

PLSA applications

- Topic modeling approach can be used for
    - Interpreting content of corpora
    - Clustering documents, predicting topics
    - Time series/trend analysis

Interpreting content of corpora [Mei et al. 07]

- How do users interpret a learned topic?
  - Human generated labels, but cannot scale up

- What makes a good label?
  - Semantically close (relevance)
  - Understandable – phrases?
  - High coverage inside topic
  - Discriminative across topics

government 0.3
response 0.2
...

city 0.2
new 0.1
orleans 0.05
...

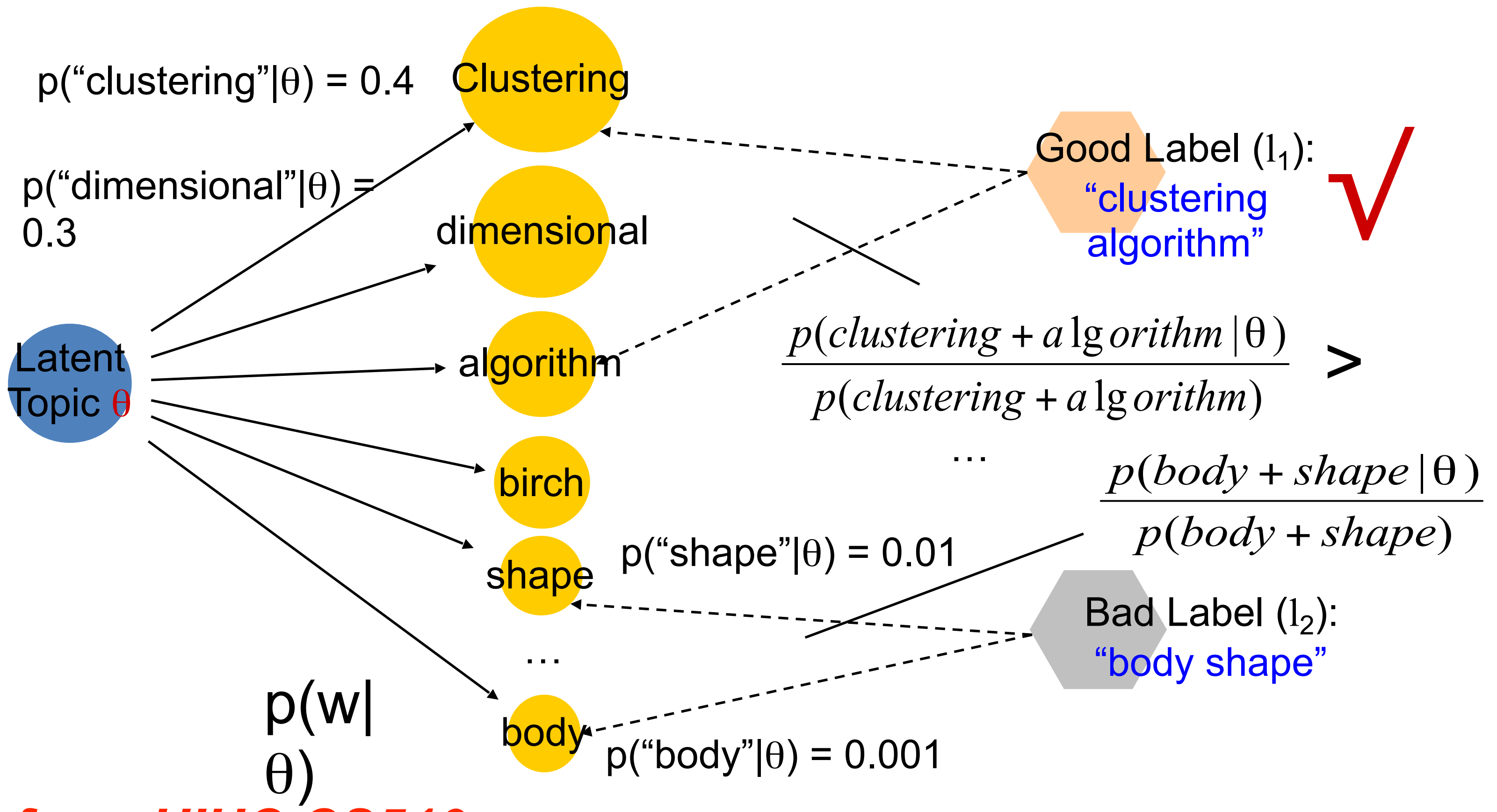
donate 0.1
relief 0.05
help 0.02
...

the 0.04
a 0.03
...

Slides from UIUC CS510

Relevance: the Zero-Order Score [Mei et al. 07]

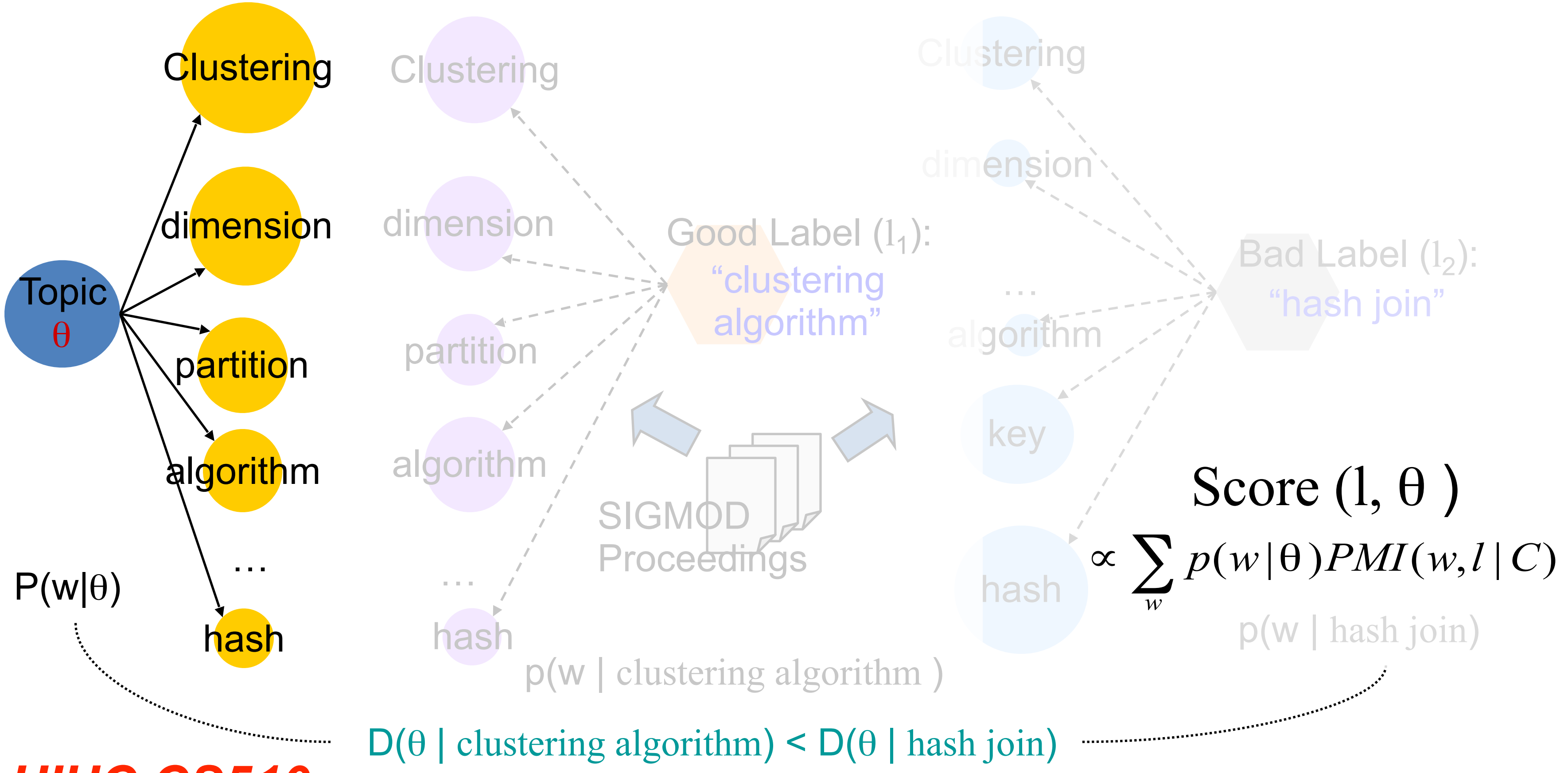
- Intuition: prefer phrases well covering top words



Slides from UIUC CS510

Relevance: the First-Order Score [Mei et al. 07]

- Intuition: prefer phrases with similar context (distribution)



$$\text{Score}(l, \theta) \propto \sum_w p(w|\theta) PMI(w, l \mid C)$$

Topic labels [Mei et al. 07]

sampling	0.06
estimation	0.04
approximate	0.04
histograms	0.03
selectivity	0.03
histogram	0.02
answers	0.02
accurate	0.02

selectivity estimation …

clustering algorithm
clustering structure
…

large data, data quality, high data, data application, …

the, of, a, and, to, data,	> 0.02
…	
clustering	0.02
time	0.01
clusters	0.01
databases	0.01
large	0.01
performance	0.01
quality	0.005

north	0.02
case	0.01
trial	0.01
iran	0.01
documents	0.01
walsh	0.009
reagan	0.009
charges	0.007

iran contra …

r tree
b tree …

indexing methods

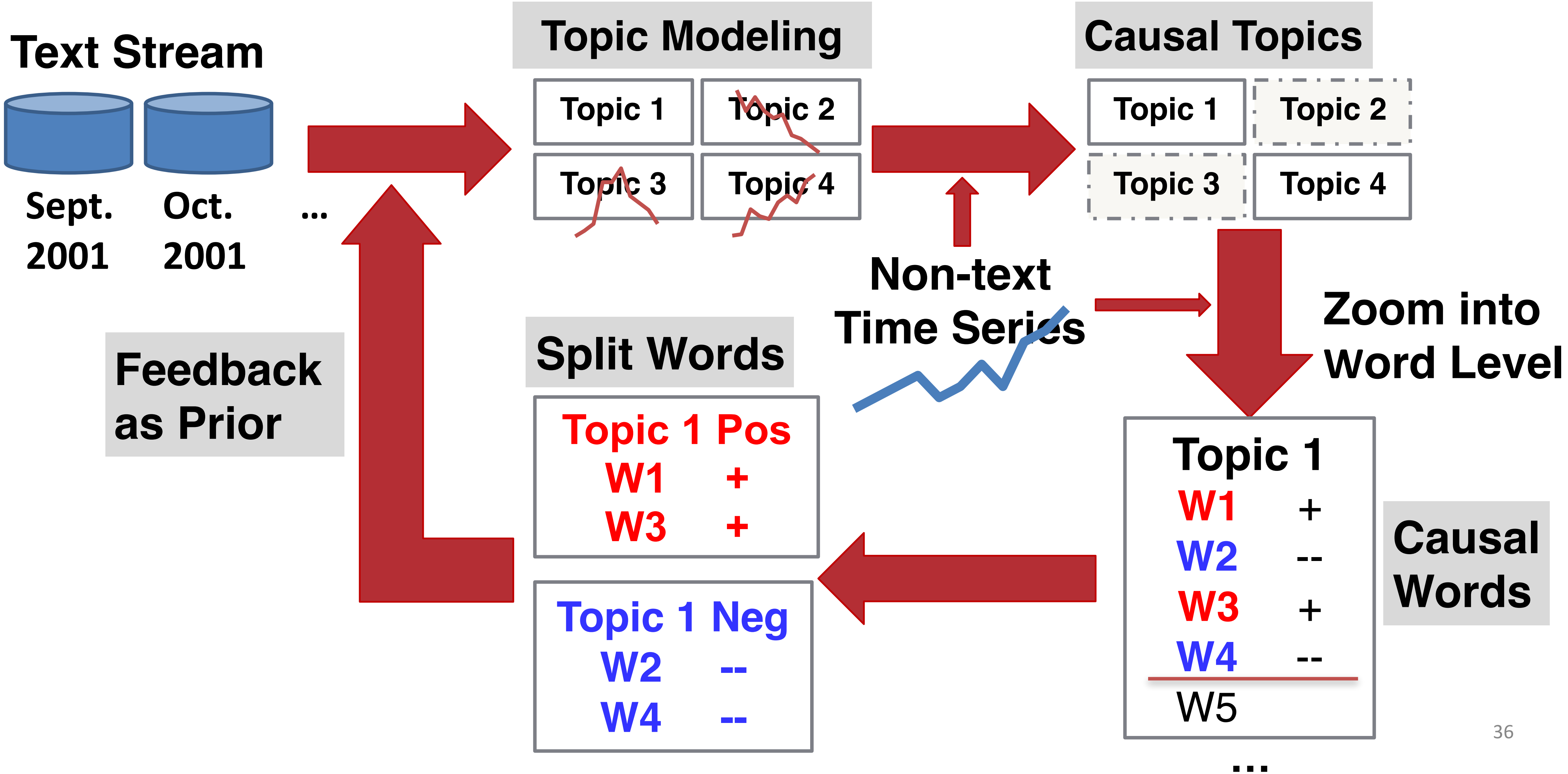
tree	0.09
trees	0.08
spatial	0.08
b	0.05
r	0.04
disk	0.02
array	0.01
cache	0.01

Text mining for understanding time series

Any clues in the companion news stream?

Dow Jones Industrial Average [Source: Yahoo Finance]

Iterative Causal Topic Modeling [Kim et al. 13]



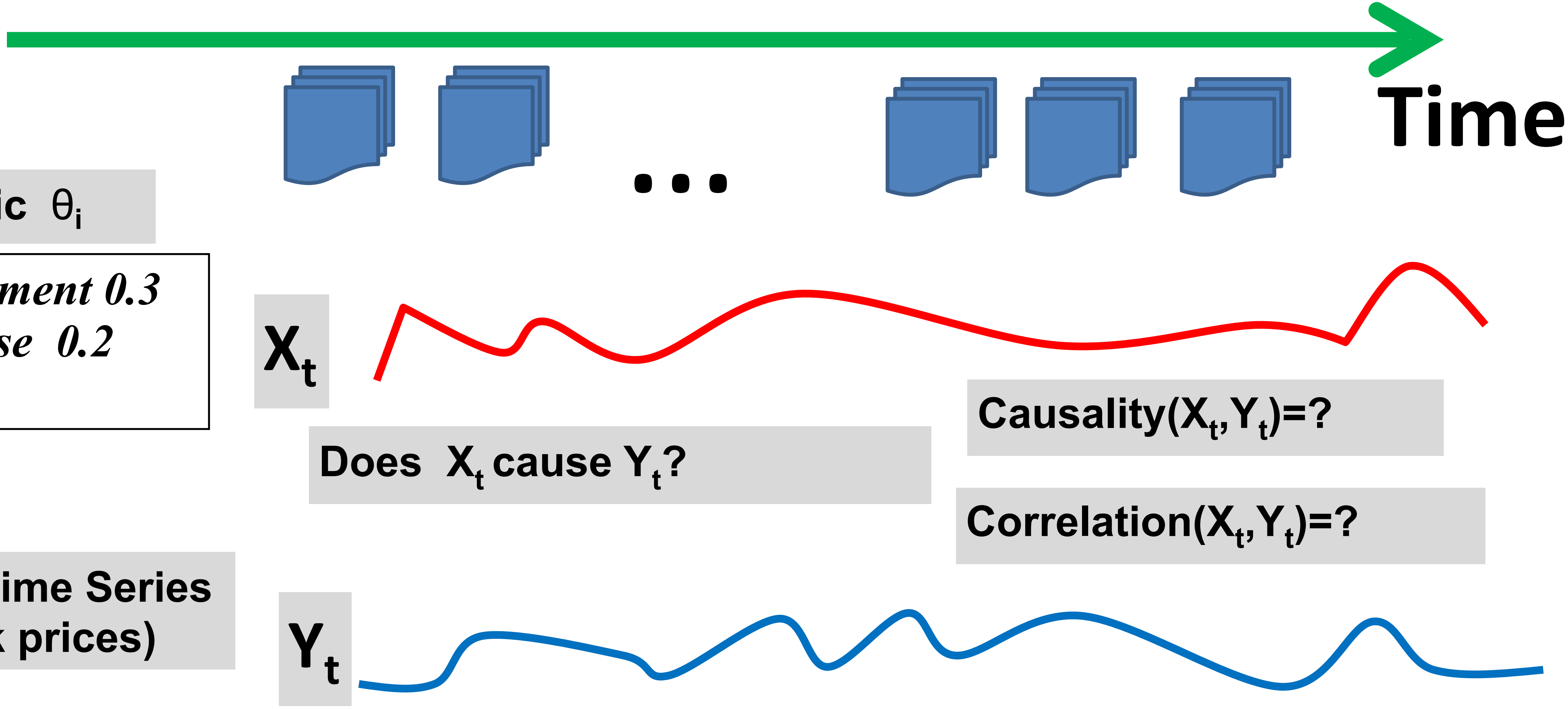
Measuring Causality (Correlation)

Topic θ_i

government 0.3
response 0.2
…

…

External Time Series
(e.g. stock prices)



Granger Causality Test is often useful [Seth 07]

Topics in NY Times Correlated with Stocks [Kim et al. 13]: June 2000 ~ Dec. 2011

AAMRQ (American Airlines)	AAPL (Apple)
russia russian putin	paid notice st
europe european germany	russia russian europe
bush gore presidential	olympic games olympics
police court judge	she her ms
airlines airport air united trade terrorism	oil ford prices
food foods cheese	black fashion blacks
nets scott basketball	**computer technology software internet com web**
tennis williams open	football giants jets
awards gay boy	japan japanese plane
moss minnesota chechnya	

Topics are biased toward each time series

Major Topics in 2000 Presidential Election [Kim et al. 13]

Top Three Words in Significant Topics from NY Times
tax cut 1
screen pataki guiliani
enthusiasm door symbolic
oil energy prices
news w top
pres al vice
love tucker presented
partial **abortion** privatization
court supreme **abortion**
gun control nra

Text: NY Times (May 2000 - Oct. 2000)

Time Series: Iowa Electronic Market
http://tippie.uiowa.edu/iem/

Issues known to be important in the 2000 presidential election

Summary

- Maximum likelihood estimation
- Expectation maximization
  - Coin-topic problem
  - Using EM algorithm to remove stop words
- Mixture of topic models
  - Probabilistic latent semantic analysis
  - PLSA with partial labels